

Determining the pressure and its distribution in solid-phase process vessels operating at high pressures requires the solution of an axisymmetric problem concerning the plastic equilibrium of a layer of a strain-hardening material compressed between shaped anvils. An exact solution to the problem of the plastic equilibrium of a layer between anvils exists for the simplest case: plane compression of the layer between flat slabs that are parallel [1] or inclined relative to one another [2, 3]. In other cases, the problem can be solved only by numerical methods. Due to the highly conditional nature of the initial assumptions made in the calculations (such as those regarding the character of the strain-hardening material) and the impossibility of accounting for all of the factors which influence the process by which pressure is created in the vessel, calculated results are presently only of an approximate nature. Thus, the use of numerical methods cannot really be considered expedient in the given case.

Even for the simplest case involving the problem of the compression of a layer between parallel slabs, the approximate solution obtained by Il'yushin [4] is widely used in engineering practice - despite the fairly simple form of the exact solution obtained by Prandtl. These solutions had existed independently of one another, but an analytical relationship established between them in [5] showed that the approximate solution is exact for the middle plane of a compressed plastic layer. In fact, the system of equilibrium equations, comprising two nonlinear equations in the Levi form

$$\begin{aligned} \frac{\partial \sigma}{\partial x} + K \sin 2\alpha \frac{\partial 2\alpha}{\partial x} + K \cos 2\alpha \frac{\partial 2\alpha}{\partial y} &= 0, \\ \frac{\partial \sigma}{\partial y} - K \sin 2\alpha \frac{\partial 2\alpha}{\partial y} + K \cos 2\alpha \frac{\partial 2\alpha}{\partial x} &= 0 \end{aligned} \quad (1)$$

is simplified considerably for points of the middle plane in the case of a plane stress state and in practice reduces to a single simple ordinary differential equation

$$\frac{d\sigma}{dx} = \pm K \frac{d2\alpha}{dy}, \quad (2)$$

where the stress components are represented by Mohr relations in terms of hydrostatic pressure  $\sigma = (\sigma_x + \sigma_y)/2$ ;  $\alpha$  is the angle formed by the direction of the largest principal normal stress and the positive direction of the x axis;  $\sigma_x$  and  $\sigma_y$  are the normal components of stress in the rectangular coordinate system xy (with the x axis being located in the middle plane of the plastic layer relative to its thickness and with its positive direction coinciding with the direction of flow of the layer material); K is the plastic constant of this material.

We write the relation  $2\alpha(y)$  as follows by making use of the Mohr relation for  $\tau_{xy}$  and the linear thickness change of the shear component  $\tau_{xy}$  established in Prandtl's exact solution [1] for the problem with parallel plates

$$2\alpha(y) = -\arcsin \frac{2y}{H} m$$

(H is the thickness of the layer being compressed;  $m = \tau_c/K$  is the ratio of the stress from contact friction to the yield strength of the material in shear). Then Eq. (2) takes the form

$$\frac{d\sigma}{dx} = \pm K \frac{2}{H} m,$$

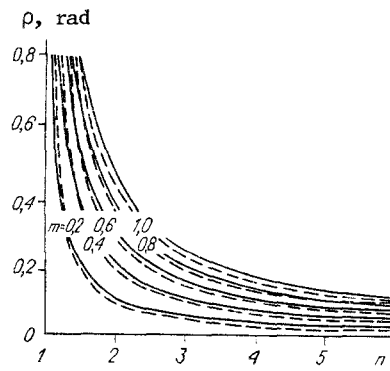


Fig. 1

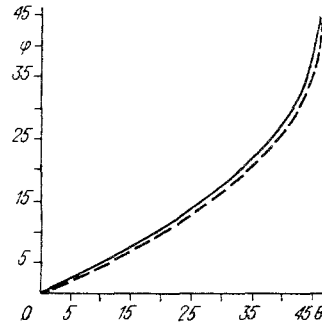


Fig. 2

which at  $m = 1$  is equivalent to the equilibrium equation in Il'yushin's approximate solution for this problem.

It is in fact the ease with which the exact solution is obtained in the middle plane that underlies the following attempt to calculate pressure in solid-phase process vessels. We take the equilibrium equation (2) for the middle plane of the plastic layer as the initial equation. This equation is valid for a layer having a cross section of arbitrary form which is symmetrical relative to the middle plane. Knowing the law governing the change in  $2\alpha$ , we can obtain a relatively simple equilibrium equation for the middle plane with a single unknown  $\sigma$ . The form of the function  $2\alpha(y)$  in the present study is determined from an analysis of well-known exact solutions of the problem of the equilibrium of a layer between anvils. If we take the contact surfaces of the anvils as envelopes to the slip lines in the plastic layer and compare the positions of tangents to these lines and the slip lines on the contact surfaces with a fixed longitudinal coordinate, we can easily determine the range  $\Delta$  of the doubled angle  $2\alpha$  of inclination of the principal stress through the layer thickness from the middle plane to the contact surface. This range is represented as the sum

$$\Delta = -(\arcsin m \pm \rho - 2\beta), \quad (3)$$

where  $\rho$  is the angle of internal friction of the layer material;  $\beta$  is the angle of inclination of a tangent to the profile of the contact surface in the positive direction of the  $x$  axis.

It can be seen from Eq. (3) that the interval  $\Delta$  includes information on contact friction, the internal friction of the material, and the form of the anvils (the angle  $\beta$  of inclination of the surface of the anvils), i.e., it contains information on the main factors which determine pressure and its distribution in the vessel. The doubled angles in the middle plane and on the contact surface ( $2\alpha_{md}$  and  $2\alpha_c$ ) are connected by the obvious relation  $2\alpha_c = 2\alpha_{md} + \Delta$ .

The quantity  $2\alpha_{md}$  can take values of 0 and  $\pi$  in the middle plane, depending on the character of flow of the layer between the anvils. For flow of the material between converging anvils,  $2\alpha_{md} = \pi$  (active flow). When the anvils are moved apart by the plastic material as it is forced into the gap between them,  $2\alpha_{md} = 0$  (passive flow). The value of the doubled angle  $2\alpha(y)$  for points of the layer not on the middle plane or the contact surface will be represented (with a fixed longitudinal coordinate) in the form of the sum  $2\alpha(y) = 2\alpha_{md}(y) + f(y)$ , where  $f(y)$  is the variable component of the relation  $2\alpha(y)$ . This component takes values of 0 and  $\Delta$  in the middle plane and on the contact surface, respectively. Thus, the derivative  $\partial 2\alpha/\partial y$  is completely determined by the component  $f(y)$  ( $\partial 2\alpha/\partial y = \partial f(y)/\partial y$ ). Analogous to the representation of the interval  $\Delta$  as a sum in Eq. (3), the variable component  $f(y)$  can be written as follows:

$$f(y) = f_m(y) + f_\rho(y) + f_\beta(y) \quad (4)$$

( $f_m(y)$ ,  $f_\rho(y)$ ,  $f_\beta(y)$  are components of  $f(y)$  determined by the contact friction and internal friction of the material of the plastic layer and the form of the anvils). The individual components can be found from known exact solutions of special cases of the problem of the equilibrium of a plastic layer between anvils. In the exact solution [1] of the problem of

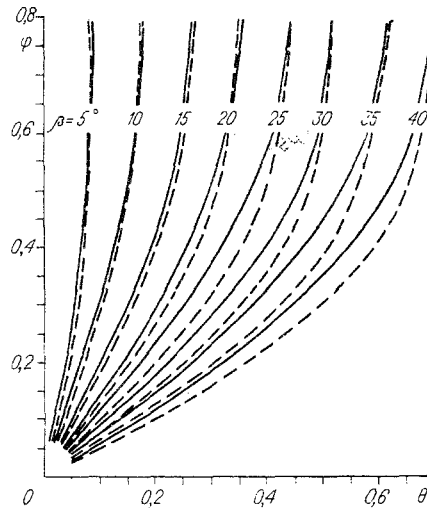


Fig. 3

the equilibrium of an ideally plastic layer between parallel slabs, the angle  $2\alpha$  is determined only by contact friction, i.e.,  $f_\beta(y) = 0$ ,  $f_p(y) = 0$  and  $f(y) = f_m(y)$ . It follows from the solution of this problem that

$$f_m(y) = -\arcsin \frac{2y}{h} m. \quad (5)$$

To check the validity of the assumption made regarding the form of the functional component  $f_m(y)$ , we analyzed the solution of the problem of the plastic radial flow of a layer in a convergent channel.

By solving the problem in a polar coordinate system  $(r\theta\phi)$  whose origin coincided with the vertex of the angle between the slabs and in which the direction  $\theta = 0$  coincided with the bisector of the angle  $2\beta$  between the slabs, the authors of [2, 3] isolated the function  $\phi(\theta)$  characterizing the change produced by contact friction in the angle between the principal stress and the  $r$  axis.

Let us compare this relation with proposed relation (5). Using the usual representation of the stress components  $\sigma_r$ ,  $\sigma_\theta$  and  $\tau_{r\theta}$  in terms of the angle  $\phi$  ( $\sigma_r = \sigma + K \cos 2\phi$ ,  $\sigma_\theta = \sigma - K \cos 2\phi$ ,  $\tau_{r\theta} = K \sin 2\phi$ ,  $\sigma = (\sigma_r + \sigma_\theta)/2$ ) and adopting the plasticity condition  $(\sigma_r - \sigma_\theta)^2/2 + \tau_{r\theta}^2 = K^2$ , we write the system of equilibrium equations in the form

$$\frac{r}{2K} \frac{\partial \sigma}{\partial r} + \cos 2\phi \left( \frac{\partial \phi}{\partial \theta} + 1 \right) = 0, \quad \frac{1}{2K} \frac{\partial \sigma}{\partial \theta} + \sin 2\phi \left( \frac{\partial \phi}{\partial \theta} + 1 \right) = 0, \quad (6)$$

and we write its solution as

$$\sigma = 2K \left[ n \ln \frac{a}{r} - \omega(\theta) \right] + K. \quad (7)$$

Inserting  $\sigma$  into Eq. (6), we obtain the equilibrium equations derived by Nadai

$$\frac{d\phi}{d\theta} = \frac{n}{\cos 2\phi} + 1, \quad \frac{d\omega}{d\theta} = n \operatorname{tg} 2\phi, \quad (8)$$

which allow us to determine  $n$  and  $\omega$ .

The solution of the first equation of (8) in [2, 3] is given in the form

$$\theta = \frac{n}{\sqrt{n^2 - 1}} \operatorname{arctg} \left( \sqrt{\frac{n+1}{n-1}} \operatorname{tg} \phi \right) - \phi. \quad (9)$$

The parameter  $n$  is connected with the angle  $2\beta$  between the slabs and with contact friction on the slabs by the relation

$$\beta = \frac{n}{\sqrt{n^2 - 1}} \operatorname{arctg} \left( \sqrt{\frac{n+1}{n-1}} \operatorname{tg} \delta \right) - \delta, \quad (10)$$

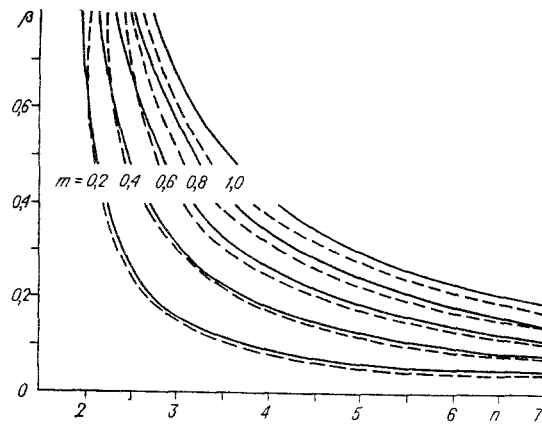


Fig. 4

where  $\sin 2\delta = \tau_c/K = m$ . Due to the impossibility of explicitly expressing the parameter  $n$  through the angle  $2\beta$  and contact friction  $m$ , the authors of [2, 3] represented Eq. (10) graphically for discrete values of  $m$  (dashed lines in Fig. 1). The solid lines show the dependence of  $n$  on  $\beta$  for the same values of  $m$  with the assumption that  $\phi$  changes through the thickness of the layer in accordance with Eq. (5). The function  $\phi(\theta)$  corresponding to (5) has the form

$$\varphi(\theta) = -\frac{1}{2} \arcsin\left(m \frac{\operatorname{tg} \theta}{\operatorname{tg} \beta}\right). \quad (11)$$

Figure 2 shows the relation  $\phi(\theta)$  in accordance with the above assumption (solid curve) and in accordance with the solution in [2, 3] (dashed curve). Inserting  $\phi(\theta)$  (11) into the first equation of system (8) and considering that  $\theta = 0$  and  $\phi = 0$  in the middle plane, we obtain

$$n = \frac{m}{2 \operatorname{tg} \beta} + 1. \quad (12)$$

It can be seen from Fig. 1 that exact relation (10) (dashed curves) and relation (12), corresponding to the given assumption (solid curves), nearly coincide for  $m < 0.8$ . The largest difference is seen at  $m = 1$  and is no greater than 5%.

The correctness of the assumption made regarding the character of the change in the component  $f_m(y)$  through the thickness of the plastic layer was similarly checked for axisymmetric radial friction in convergent channels (this problem was solved in [2]). The solution was obtained in spherical coordinates.

The stress components  $\sigma_r$ ,  $\sigma_\theta$ ,  $\sigma_\varphi$ ,  $\tau_{r\theta}$  were represented through the angle  $\phi$  by the usual relations

$$\begin{aligned} \sigma_r &= \sigma + \frac{2}{\sqrt{3}} K \cos \varphi, & \sigma_\varphi = \sigma_\theta &= \sigma - \frac{1}{\sqrt{3}} K \cos \varphi, \\ \tau_{r\theta} &= K \sin 2\varphi, & \sigma &= \frac{\sigma_r + \sigma_\theta + \sigma_\varphi}{3}. \end{aligned}$$

The plasticity condition was taken in the form  $3/4(\sigma_r - \sigma_\theta)^2 + \tau_{r\theta}^2 = K^2$ . The differential equations of equilibrium in spherical coordinates

$$\begin{aligned} \frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{1}{r} [2\sigma_r - (\sigma_\theta + \sigma_\varphi) + \tau_{r\theta} \operatorname{ctg} \theta] &= 0, \\ \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{1}{r} [3\tau_{r\theta} + (\sigma_\theta - \sigma_\varphi) \operatorname{ctg} \theta] &= 0 \end{aligned}$$

take the following form after the relations for the stress components are inserted into them

$$\begin{aligned} \frac{r\partial\sigma}{2K\partial r} + \cos 2\varphi \left( \frac{\partial\varphi}{\partial\theta} + \sqrt{3} \right) + \frac{1}{2} \operatorname{ctg} \theta \sin 2\varphi &= 0, \\ \frac{1}{2} \frac{\partial\sigma}{K\partial\theta} + \sin 2\varphi \left( \frac{1}{\sqrt{3}} \frac{\partial\varphi}{\partial\theta} + \frac{3}{2} \right) &= 0. \end{aligned} \quad (13)$$

As in the case of the plane problem,  $\sigma$  is represented in the form

$$\sigma = 2K \left[ n \ln \frac{a}{r} - \omega(\theta) \right] + \frac{K}{\sqrt{3}} \quad (14)$$

With  $\theta = 0$ ,  $\omega(\theta) = 0$ . After we insert (14) into (13) we obtain differential equations for  $n$  and  $\omega$ :

$$\frac{d\varphi}{d\theta} = \frac{n}{\cos 2\varphi} - \frac{1}{2} \operatorname{ctg} \theta \operatorname{tg} 2\varphi - \sqrt{3}, \quad \frac{d\omega}{d\theta} = \sin 2\varphi \left[ \frac{1}{\sqrt{3}} \frac{d\varphi}{d\theta} + \frac{3}{2} \right]. \quad (15)$$

Only the results of numerical solution of this system were presented in [2, 3].

If we assume that the change in the angle  $\phi$  through the layer thickness - dependent only on contact friction on the slabs (as in the plane problem of flow between parallel or inclined slabs) - is determined by a single relation  $\phi(\theta) = -(1/2) \arcsin(m(\tan\theta/\tan\beta))$ , then after we insert it into the first equation of system (15) we obtain a simple expression for  $n$  as a function of the angle  $\beta$  between the generatrices of the channel and the friction coefficient on them:

$$n = \frac{m}{2 \operatorname{tg} \beta} + \sqrt{3}. \quad (16)$$

Figure 3 shows values of  $\phi(\theta)$  from the numerical solution of system (15) (dashed lines) and from the use of assumption (10) (solid lines) for  $m = 1$  and  $\beta$ . Figure 4 shows the dependence of  $n$  on the angle  $\beta$  between slabs for  $m$  in accordance with the results of the numerical solution of system (15) (dashed curves) and approximation (16) (solid curves). It can be seen that the difference between the approximate and exact solutions is no greater than 10%.

Proceeding on the basis of the above comparison, we write the function for the change in the angle  $f(y)$  through the layer thickness for an ideally plastic layer as  $f(y) = 2\phi(\theta) + 2\theta$ . It follows from the equation  $2\phi(\theta) = f_m(y)$  that  $2\theta = f_\beta(y)$ . The final form of the functional component  $f_\beta(y)$  is determined from the relations between the polar and rectangular coordinates:  $2\theta = \arctan(2y/H \operatorname{tg}\beta)$ , for an ideally plastic layer

$$f(y) = - \left[ \arcsin \left( \frac{2y}{H} m \right) - 2 \operatorname{arctg} \left( \frac{2y}{H} \operatorname{tg} \beta \right) \right]. \quad (17)$$

The equation of equilibrium of a plastic layer in the middle plane takes the form

$$\frac{\partial\sigma}{\partial x} = \pm \frac{2}{H} K (m - 2 \operatorname{tg} \beta).$$

The character of the change in the third term  $2\alpha(y)$  in Eq. (4) can be determined from the solution of the Hartman problem on the compression of a layer of a generalized plastic material ( $\rho \neq 0$ ) between parallel slabs [6]. In this case, the component  $f(y)$  depends only on the contact friction on the slabs and the internal friction of the material. Knowing the character of the functional component  $f_m(y)$  determined by contact friction, we can use the solution of the problem to find the character of the second functional component  $f_\rho(y)$  determined by the internal friction of the material. Based on analysis of the solution of the Hartman problem, the derivative is represented in the form

$$\frac{\partial f_\rho}{\partial \left( \frac{y}{H/2} \right)} = \rho (1 - 0.5 m^2). \quad (18)$$

By finding the value of the derivative in the above equation, we can determine the stress state of a strain-hardening plastic layer between rough anvils of arbitrary form. Such a solution was used to find pressure and its distribution in a certain variant of a

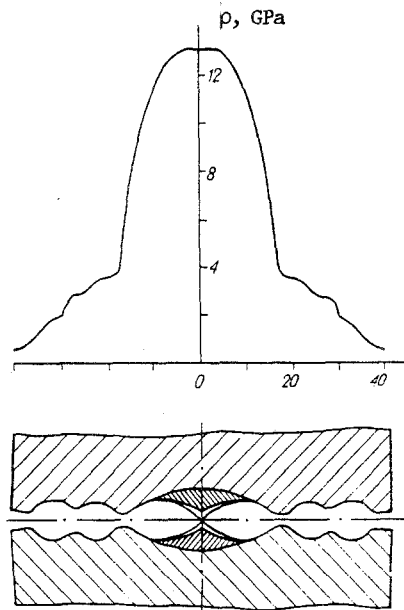


Fig. 5

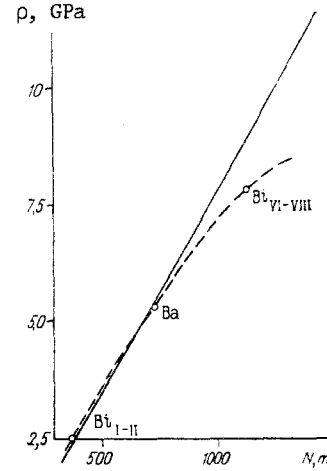


Fig. 6

solid-phase process vessel [7] with shaped anvils. Figure 5 presents a schematic diagram of the working zone of the vessel (lower part). Central spherical depressions and two rows of concentric depressions with a smooth profile were made on the ends of the anvils. The profile of the projections on the working surface of the anvils is described by arcs of radius  $r_1$ , while the profile of the depressions is described by arcs of radius  $r$ . The working container and compression rings, made of a material with the internal friction  $\rho$ , are located between the shaped anvils in the central and annular depressions, respectively. Pressure is created with the compression of the container between the working ends of the anvils by means of the set of compression rings. Finding the pressure in the vessel reduces to determination of the stress state of the container and the compression rings. The problem was solved in a cylindrical coordinate system  $r\theta z$ . Here, the  $z$  axis coincides with the direction of convergence of the dies, while the plane  $r\theta$  coincides with the middle plane of the container and the compression rings. The material of the container and the compression rings was assumed to be a generalized ideally plastic material having a shear strength that obeys Coulomb's law. If we assume that this law is linear in character, i.e., if we assume that the envelope of the Mohr's circles is a straight line, then the plasticity condition can be represented in the form

$$K = K_0 \cos \rho + \sigma \sin \rho, \quad (19)$$

where  $\sigma = (\sigma_r + \sigma_z)/2$  is the mean stress in the plane of the axial section. The components  $\sigma_r$ ,  $\sigma_z$ ,  $\tau_{rz}$  of the stress state in the meridional plane  $rz$  will be represented by Mohr relations as a function of the mean stress  $\sigma$  ( $\sigma < 0$ ) in this plane and the angle  $\alpha$  of inclination of the greatest principal stress in the positive direction of the  $r$  axis:

$$\sigma_r = \sigma + K \cos 2\alpha, \quad \sigma_z = \sigma - K \cos 2\alpha, \quad \tau_{rz} = K \sin 2\alpha. \quad (20)$$

Adopting the Harr-Karman condition for complete plasticity  $\sigma_\theta = \sigma_1 = \sigma_2$  and assuming a plastic flow regime corresponding to the edge of a Tresca prism

$$\sigma_2 - \sigma_3 = 2K, \quad (21)$$

we obtain the following for the hoop stress

$$\sigma_\theta = \sigma + K. \quad (22)$$

With allowance for Eqs. (19)-(22), the system of differential equilibrium equations in the cylindrical coordinate system  $r\theta z$

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + (\sigma_r - \sigma_\theta)/r = 0, \quad \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \tau_{rz}/r = 0$$

can be represented in the form

$$\begin{aligned} \frac{\partial \sigma}{\partial r} &= K \sin 2\alpha \frac{\partial 2\alpha}{\partial r} - K \cos 2\alpha \frac{\partial 2\alpha}{\partial z} + K(1 - \cos 2\alpha)/r - \frac{\partial K}{\partial r} \cos 2\alpha - \frac{\partial K}{\partial z} \sin 2\alpha, \\ \frac{\partial \sigma}{\partial z} &= \frac{\partial K}{\partial z} \cos 2\alpha - \frac{\partial K}{\partial r} \sin 2\alpha - K \sin 2\alpha \frac{\partial 2\alpha}{\partial z} - K \cos 2\alpha \frac{\partial 2\alpha}{\partial r} - K \sin 2\alpha/r \end{aligned}$$

and, for the middle plane, can for practical purposes be reduced to a single equation

$$\frac{\partial(\sigma + K)}{\partial r} = -K \frac{\partial 2\alpha}{\partial z}. \quad (23)$$

With allowance for (4), (17), and (18), Eq. (23) takes the form

$$\frac{\partial(\sigma + K)}{\partial r} = -\frac{K}{H(r)/2} [m - 2 \operatorname{tg} \beta + \rho(1 - 0.5 m^2)], \quad (24)$$

where  $H(r)/2$  (the running thickness of the compressed rings and container) depends on the profile of the anvils;  $\operatorname{tg} \beta = (d(H(r)/2)/dr)$ . The final form of Eq. (24) is determined by the form of the anvils (the relation  $H(4)$ ).

In the solution of Eq. (23), the normal and shear stresses on the free surfaces are taken equal to zero. Two regions are distinguished in the compressed materials of the rings and container: the outer region, in which all of the material between the anvils is assumed to be in the plastic state (and contact friction on the surfaces of the anvils is assumed to be equal to the ultimate strength of the compressed material in shear ( $m = 1$ )); a central region in which only that portion of the compressed material adjacent to its middle plane is in the plastic state. The boundary separating the rigid and plastic regions usually serves as the slip line, which is drawn from the coordinate origin to the contact surface of the anvils. To simplify the solution of Eq. (24), the region of the plastic state in the central zone was delimited by a circle arc passing through the origin so that a tangent to the arc formed the angle  $\pi/4 + \rho/2$  with the  $r$  axis (as the slip line). The plastic region in the central zone was also bound by a tangent to the anvil profile at the point  $r = r_0$  (determined, as in [5], from the mass balance). The shear stresses on the boundary between the rigid and plastic zones of the central region were taken equal to the ultimate strength of the container material in shear.

Although the difference between the rigid and plastic zones of the central region (bounded by the slip line and the circle arc) was almost negligible, replacing the slip line by a circle arc appreciably simplified the solution of differential equilibrium equation (24).

In the case when the plastic zones are bounded by circle arcs or segments of straight lines, numerical integration of Eq. (24) is unnecessary. Given the above assumptions, we use the formula for the integral of Eq. (24) to perform calculations over the entire interval of  $r$  from 0 to the free boundary. The parameters characterizing the properties of the material of the compression rings and container were taken from data in [8]. In the center of the vessel (with small  $r$ ), the angle of inclination of a tangent to the boundary between the rigid and plastic zones increases to values at which the pressure gradient in (24) becomes equal to 0 and subsequently changes sign, i.e., this angle increases to values at which a trough appears on the curve describing the pressure distribution at the center of the middle plane. From the viewpoint of flow kinematics, the change in the sign of the pressure gradient means that there is a change in the direction of flow of the material. Since we did not allow for all of the factors that might have influenced the flow direction in the compressed material (the anvils were assumed to be rigid, the materials of the container and compression rings were assumed to be incompressible and nonporous), the compressed material should have flowed in only one direction - from the center to the periphery of the vessel over the entire interval of  $r$  from 0 to its free boundary. The change seen in the sign of the gradient  $d\sigma/dr$  in the center of the vessel is connected with the fact that the theoretical boundary of the plastic zone at the center of the vessel does not coincide with

the actual boundary, i.e., in accordance with Eq. (24), the angle of inclination of the tangent to the boundary of the plastic zone cannot exceed the value

$$2 \operatorname{tg} \beta \geq m + \rho(1 - 0.5m^2). \quad (25)$$

Figure 5 shows the boundary of the plastic zone at the center of the vessel when corrected with allowance for condition (25). Instead of being bounded by the slip line or the substitute circle arc, this zone is delimited by a tangent to it drawn at an angle in accordance with condition (25). The presence of a conical plastic region at the center of the vessel means that there is a nongradient pressure region at this location. This has been confirmed by numerous experiments. The new boundaries of the plastic zones preclude point contact with the rigid zones located on opposite sides of the middle plane of the container. This situation leads to nearly infinite theoretical values of the stress components at the center of the vessel [9]. Figure 5 shows the pressure distribution in the middle plane of the compressed rings and container.

We used the distribution of the component  $\sigma_z$  in the middle plane to find the force and construct the calibration curve  $N = f(p_0)$ , where  $p_0 = (\sigma_r + \sigma_\theta + \sigma_z)/3$  is the pressure at the center of the vessel. Figure 6 shows experimental results and results from calculation of the function  $N = f(p_0)$  for a variant of the vessel in [7] with a central hole 35 mm in diameter.

#### LITERATURE CITED

1. L. Prandtl, "Examples of application of the Hencky theorem to the equilibrium of plastic bodies," in: Theory of Plasticity [Russian translation], IL, Moscow (1948).
2. V. V. Sokolovskii, "Plane and axisymmetric equilibrium of a plastic mass between rigid walls," Prikl. Mat. Mekh., 14, No. 1 (1950).
3. V. V. Sokolovskii, "Plane equilibrium of a plastic wedge," ibid., 14, No. 4 (1950).
4. A. A. Il'yushin, "Problems of the theory of the flow of a plastic substance over surfaces," ibid., 18, No. 3 (1954).
5. N. N. Kuzin, Yu. A. Sadkov, and A. A. Semerchan, "Calculation of calibration curves of pressure vessels with shaped anvils" Prikl. Mekh. Tekh. Fiz., No. 3 (1976).
6. A. Nadai, Plastic Deformation and Fracture of Solids [Russian translation], IL, Moscow (1954).
7. L. F. Vereshchagin, A. A. Semerchan, N. N. Kuzin, and Yu. A. Sadkov, "Data on the performance of a three-stage pressure vessel with a working volume of 100 cm<sup>3</sup>," Dokl. Akad. Nauk SSSR, 202, No. 1 (1972).
8. L. F. Vereshchagin and E. V. Zubova, "Measurement of shear stresses in several substances at pressures up to 100,000 atm," Dokl. Akad. Nauk SSSR, 134, No. 4 (1960).
9. N. V. Novikov and V. I. Levitas, "Solution of an axisymmetric problem on the plastic flow of material in certain high-pressure chambers," Sver. Mater., No. 5 (1980).